SOME CONJECTURES ON "COMMODITY TECHNOLOGY" ASSUMPTION

福 井 幸 男
By Yukio Fukui*

1 Introduction

The new System of National Accounts (SNA), which was proposed in 1968 by the United Nations, includes a general input–output accounting framework. Use table $U=(u_{ij})$ of commodities i consumed by industries j and make table $V=(v_{ij})$ of industries i producing commodities j are its main two tables. But it does not contain an input–output table $A=(a_{ij})$ of commodities i for commodities j. So, the SNA input–output framework is not directly useful for any type of conventional input–output analysis.

To convey the information as presented in this framework, the SNA suggests conversion techniques based on alternative set of assumptions.

First is the industry technology assumption, which supposes that the industries are homogeneous as far as production techniques are concerned. This implies that the inputs of industries are proportional to their total outputs. Second is the commodity technology assumption, which supposes that technology is intrinsic to the commodity wherever it is produced, implying the inputs of commodities are stable and unique.

Which assumption to choose, including all variants of the two assumptions, has been considered by, among others, Stone, Bates and Bacharach (1963), ten Raa, Chakraborty and Small (1984), and Gigantes (1970). ten Raa et al critised the industry technology assumption on its sensitivity to base–year prices.[2] McGilvray and Morrison (1982), on the other hand, took the assumption to be more preferable for a practical reason, writing that "the industry technology assumption enables a more detailed classification of commodities than industries to be used" (p. 246). Olsen (1984) and Thage (1982) hold the same view on this point.

Since we are interested in computational problems, we focus our attention on the commodity technology assumption. By setting Ac as a commodity–by–commodity table under the assumption, the use table can be written as $U=Ac\,V'$. Furthermore, with $g=V1$, we define the two matrices:

$B=U\hat{g}^{-1}$ matrix of industry input coefficients

$C=V'\hat{g}^{-1}$ matrix of industry products–mix

Here $\hat{g}$ denotes a diagonal matrix with the elements of g in the diagonal.

Then we have

$$Ac=BC^{-1} \qquad (1).$$

While it is well known that the commodity technology assumption will yield negative

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* The Department of Economics, Osaka Industrial University.

coefficients (e. g. United Nations (1968, Chapter 3) and Stone, Bates and Bacharach (1963, pp. 16−18)) we stress however that the assumption has potentially a more serious shortcoming; it is likely to yield economically meaning less negative column sums. Although the Stone method (Stone (1961, pp. 39−41)), in which joint products are handled to be minus inputs, will yield negative coefficients, his procedure is commonly used in practice in Japan, for example (Administrative Management Agency (1984, p. 22)).

Therefore Ac with some negative entries does not discourage us from using it as a tool for applied input–output analysis. However, to be economically meaningful, all the column sums must be nonnegative; otherwise they imply the instability of economic systems.

Section 2 introduces the definition of dominant diagonal of C, and attacks the commodity technology assumption on computational grounds theoretically. Section 3 evaluates the drawback by simulation, and presents three conjectures of the possibility of column sums being under zero. This section also illustrates examples. Section 4 concludes the paper.

2 Theoretical View

Consider the static input output system: $y=(I-A)^{-1}f$ where y is the n–length column vectors $y=\{y_i\}$ and $f=\{f_i\}$, and A is the $n \times n$ matrix, with $A \geqq 0$. Here, y_i is the total output of commodities y_i, and f_i is the final demand for commodities i. Notice that for these steps to be meaningful for a specified $f \geqq 0$, $(I-A)^{-1}$ must exist and this inverse must be such that $y \geqq 0$. A necessary and sufficient condition for $(I-A)^{-1} \geqq 0$ is that all principal minors of A be positive (Hawkins and Simon (1952)). Further, the Solow condition on column sums of A is a sufficient condition for it. Notice that even if column sums of A are over unity, it does not necessarily mean the instability of economic systems. If the column sums are under zero, however, it means the instability.

Before we prove the very strong possibility of some column sums of A being under zero, we impose the following assumption on the matrix of industry commodity shares C.

ASSUMPTION: *C is dominant diagonal*

Here, $C=(c_{ij})$, i, j=1, . . , n, is said to have a dominant diagonal (Taussky (1949, p. 672)) if

$$|c_{jj}| > C_j \qquad \text{for all } j$$

with $C_j=\Sigma_{k \neq j} |c_{kj}|$. The associated sums are:

$$M_j=c_{jj}+\Sigma^{n}_{k\ \ j+1} c_{kj} \qquad \text{for } j=1, . . . , n-1$$

$$m_j=c_{jj}-\Sigma^{n}_{k=j+1} c_{kj} \qquad \text{for } j=1, . . . , n-1$$

$$M_j{*}=\Sigma^{n}_{k=1} c_{kj} \qquad \text{for all } j$$

$$m_j{*}=c_{jj}-\Sigma^{n}_{k \neq j} c_{kj} \qquad \text{for all } j \tag{2}$$

First we discuss our assumption. This is not a strong assumption (Divay (1982, p. 194)) when each industry concentrates its characteristic products, which is always the case.

We now state two lemmas for later use.

LEMMA 1 (Taussky (1949, pp. 672−673)): *Let $C=(c_{ij})$, i, j=1, . . , n be a real matrix which has a dominant diagonal. Then C is non–singular.*

(Proof) Assume the contrary. Since C is singular, it follows that the system of equations

$$\Sigma_{k=1}^{n} c_{kj}\, x_k = 0, \qquad \text{for all } j,$$

has a non-trivial solution $x_1, \ldots x_n$. Let r be one of the indices for which $|x_i|$, $j=1, \ldots, n$, is maximal. The rth equation implies

$$|c_{rr}|\ |x_r| \geqq \Sigma_{k\neq r}^{n} |c_{kr}|\ |x_k| \geqq C_r |x_r|,$$

where $C_r = \Sigma_{k\neq r}^{n} |c_{kr}|$. This contradicts the hypothesis. Q. E. D.

LEMMA 2 (Price (1951, p. 500)): *Let the matrix $C_{11}=(c_{ij})$, $i, j=2, \ldots, n$, denotes a dominant diagonal matrix, and*

$$c_{1j} + \Sigma_{k=2}^{n} c_{kj}\, x_k = 0, \qquad j=2, \ldots, n, \tag{3}$$

be a system of equations, and let some number $r>0$ be such that

$$|c_{jj}| > (1/r)\, |c_{1j}| + \Sigma_{k=2,\, k\neq j}^{n} |c_{kj}| \qquad j=2, \ldots, n. \tag{4}$$

If the system has a unique solution, $x_2, \ldots, x_n$, then

$$\max(|x_2|, \ldots, |x_n|) < r. \tag{5}$$

(Proof) From (4), we get

$$|c_{jj}| > \Sigma_{k=2,\ k\neq j}^{n} |c_{kj}|, \qquad j=2, \ldots, n.$$

Therefore, A is not singular by lemma 1 and (3) has a unique solution. Assume that (5) is not true. Let $|x_i| = \max(|x_2|, \ldots, |x_n|) \geqq r$. From (3) it follows that $-c_{ii}x_i = c_{1i} + \Sigma_{k=2,\ k\neq i}^{n} c_{ki}x_k$. We get by means of the elementary inequality

$$|c_{ii}|\ |x_i| \leqq |c_{1i}| + \Sigma_{k=2,\ k\neq i}^{n} |c_{ki}|\ |x_k|,$$

so that

$$|c_{ii}| \leqq (1/r)\, |c_{1i}| + \Sigma_{k=2,\ k\neq i}^{n} |c_{ki}|,$$

This contradicts (4). Q. E. D.

THEOREM 1 (Price, 1951, p. 498): *Let C be a dominant diagonal matrix. Then*

$$0 < m_1 \ldots m_n \leqq |C| \leqq M_1 \ldots M_n \tag{6}$$

(Proof) The assumption of C and $m_j>0$ for all j assure the first inequality. Consider the equation system (3) with the coefficient matrix $C_{11}=(c_{ij})$, $i, j=2, \ldots, n$, taken from C. Then the definition of C implies that (4) is satisfied with $r=1$. Let the solution of (3) be $x_2^{(1)}, \ldots, x_n^{(1)}$ Multiply the jth row of C by $x_j^{(1)}$, $j=2, \ldots, n$, add to the first row and use (3) to get

$$|C| = (c_{11} + c_{21}x_2^{(1)} + \cdots + c_{n1}x_n^{(1)})\ |C_{11}|$$

Thus the inductive reasoning follows

$$|C| = (c_{11} + c_{21}x_2^{(1)} + \cdots + c_{n1}x_n^{(1)})\ (c_{22} + c_{32}x_3^{(2)} + \cdots + c_{n2}x_n^{(2)}) \cdots c_{nn}.$$

Since $|x_j^{(k)}| < 1$ for all k and j, we get (6). Q. E. D.

The theorem contains the following corollary as a specal case.

COROLLARY 1: *If $C^{-1}=(c_{ij}*)$, $i, j=1, \ldots, n$, then*

$$1/M_i* \leqq c_{ii}* \leqq 1/m_i* \tag{7}$$

(Proof) Principal minors of C are invariant under any permutations of its same rows and columns, thus we can take $i=1$ without loss of generality. Then

$$m_1 |C_{11}| \leqq |C| \leqq M_1 |C_{11}|.$$

Since $c_{11}* = |C_{11}| / |C|$, $M_1* = M_1$ and $M_n* = M_n$, we get (7). Q. E. D.

THEOREM 2: *Let C be a $n\times n$ non-negative matrix with the same column smus λ. Then the inverse C^{-1} has the same column sum $1/\lambda$.*

(Proof) With $1=(1,\ldots,1)$, $C'1=(\lambda,\ldots,\lambda)'=\lambda 1$. Premultiplying $(1/\lambda)C'^{-1}$ gives $(1/\lambda)1=C'^{-1}1$. Q. E. D.

Since each column sum of C is unity by definition, it follows from theorem 2 that the inverse of C has the same column sums;

$$C'^{-1}1=1. \quad (8)$$

Using $M_i*=1$, (7) can be written

$$1\leqq c_{jj}*\leqq 1/m_i* \quad (9).$$

The above inequality suggests that l/m_j*, upper limit of $c_{jj}*$, tends to decrease with c_{jj}, as given in (2). Before proceeding we present the following theorem.

THEOREM 3: *Column sums of Ac are same across all commodities, if and only if all column sums of B are equal across all industries.*

(Proof) $Ac'\ 1=C'^{-1}\ B'\ 1=C'^{-1}\ s,\ =s*$ say, with $s=B'\ 1$. Premultiplying C' to the third equation, we get $s=C's*$. If $s*=\bar{s}1$, then we get immediately $s=\bar{s}1$. Conversely, substituting $s=\bar{s}\ 1$ into $s*=C'^{-1}s$ to get $s*=\bar{s}\,C'^{-1}\ 1=\bar{s}\ 1$. Last equation holds from (8). Q. E. D.

Now the *j*th column sum of Ac, given in (1), can be written by

$$s_j*=c_{jj}*s_j*+\Sigma_{i\neq j}^{n}c_{ij}*s_i \quad (10)$$

where s_i is the ith column sum of B. The crucial point is that, the second term can be greater than first term in absolute value, because $c_{jj}*\geqq 1$ and $\Sigma_i c_{ij}*=1$ imply that at least one element must be negative. More precisely, if a negative element is sufficiently large, s_j* is likely to be negative as well. The possibility will getting higher, in particular, if the corresponding s_i is also large. On the other hand, if the first term is much greater than the second, s_j* will be over unity. This is most likely to occur when the corresponding s_j is large and some s_i's $(i\neq j)$ are small.

The issues involved in analyzing these cases will be illustrated by the following simplest case, $n=2$. Then (10) can be written

$$s_1*=c_{11}*s_1+c_{21}*s_2,\ s_2*=c_{12}*s_1+c_{22}*s_2.$$

Recall that $c_{11}*+c_{21}*=1$ and $c_{12}*+c_{22}*=1$ to get

$$s_1*=s_2+c_{11}*(s_1-s_2),\ s_2*=s_1+c_{22}*(s_2-s_1).$$

These imply that $s_1*<0$ if $s_1<s_2$ and $s_2/(s_2-s_1)<c_{11}*$, $s_2<0$ if $s_1>s_2$ and $s_1/(s_1-s_2)<c_{22}*$, and $s_1*=s_2*$ only if $s_1=s_2$. Consider the example.

$$C=\begin{pmatrix}0.7 & 0.4\\ 0.3 & 0.6\end{pmatrix},\ s_1=0.8,\text{ and } s_2=0.2.$$

Then it is easy to get the inverse of C and column sums of Ac as

$$C^{-1}=\begin{pmatrix}2 & -1.3\\ -1 & 2.3\end{pmatrix},\ s_1*=1.4,\text{ and } s_2*=-0.58.$$

Note the following features of the possibilities in the example. First, a higher value of s_1 and lower value of s_2 result in higher value of s_1* and a lower value of s_2*. In the case $s_1=0.9$ and

$s_2=0.1$, we get $s_1*=1.7$ and $s_2=-0.94$ keeping c_{ij} fixed. Second, a higher value of k result in lower value of s_i* and in a higher value fo s_2*. At $c_{11}=c_{22}=0.9$, we get $s_1*=0.86$ and $s_2=0.14$ keeping s_1 and s_2 fixed. Finally, $s_1*=s_2*=0.5$ at $s_1=s_2=0.5$, coincident with theorem 3.

3 Simulation

In the previous section, under the assumption of commodity technology, we showed that some s_j'* will be possibly under zero (C0) when k is small and/or s_j is small and some s_i's ($i \neq j$) are large; conversely s_j* over unity (C1) when k is small and/or s_j is large and some s_i'* ($i \neq j$) are small. In practice, further knowledge concerning the extent to which the validity of our argument can be proved is needed. While it is not our intention to furnish a comprehensive answer for this issue, however, we hope to shed some light on it by presenting the results of a set of carefull desighed Monte Carlo experiments.

These experiments are preformed under various conditions. At first the values of c_{ij} for the C matrix were generated by the following simple scheme:

$$c_{jj}=u_{jj};\quad c_{ij}=u_{ij}\times(1-u_{ii})/\Sigma^{n}_{i\neq j}u_{ij}$$

The values of u_{ij}'s were randomly generated from a uniform distribution on [k, l], where k means the proportion of characteristic product in total value of products of each industry. The value of k must be over 0.50, which is due to the fact that C is dominant diagonal. Then the values of u_{ij} ($i \neq j$) were also generated from a distribution on [0, 1] independently of u_{jj}. These u_{ij} ($i \neq j$) were multiplied by $(1-u_{ii})/\Sigma^{n}_{i\neq j}u_{ij}$ so as to make the *j*th column sum equal to unity. Next the values of s_j, the *j*th column sum of B were also generated from a uniform distribution on [0, 1].

3–1 Effects of n and k on s_i

The first simulation is conducted to study the influence of the number of commodities and the value of k on s_i*. This was specified by the following parameter values; $k=0.5$, 0.9, and $n=4$, 22. When $n=4$, we carried out 10 experiments, each with 160 random vector s and 200 random matrix C. When $n=22$, we conducted 20 experiments, each with 10 random vector s and 20 random matrix C.

Table 1 shows the results. The first row shows that for $n=4$ and $k=0.5$, the proportions of Ac's with C0 and C1 are 49.6 and 48.1 per cent respectively. The second row shows that for $k=0.9$ the per cent is considerably smaller than the first with the same n. Notice also that the same downward tendency can be obtained for $n=22$. Both in $n=4$ and in $n=22$ the results agree with the theoretical consideration in Section 2. The following conjecture is to this effect.

CONJECTURE 1: *The more the extent that a number of commodities are produced in more than one industry as secondary products, the higher is the possibility of Ac having colums sums under zero and/or over unity.*

A comparison of the first two rows for $n=4$ and the second two rows for $n=22$ reveals substantively divergent results. According to row 1 and row 3, the possibility of C0 for $n=22$ exceeds the possibility of C0 for $n=4$ with the same k by about 50 per cent. This indicates that

TABLE 1 Simulation Results ($0 \leqq s_i \leqq 1$)*

n	k	Num of Ac	C0 (under zero)	C1 (over unity)	C0+C1	C0−C1
4	0.5	320,000	158,735[49.6%] (178,873)	154,077[48.1%] (172,245)	88,587	224,225
	0.9	320,000	29,260 [9.1%] (34,324)	27,795 [8.6%] (31,647)	3,043	54,012
22	0.5	4,000	3,877[96.9%] (11,302)	3,877[96.9%] (11,011)	3,759	3,996
	0.9	4,000	2,007[50.2%] (2,742)	1,526[38.2%] (1,684)	821	2,712

*Numbers in brackets are percentages of Ac with C0 (C1) column sums, and numbers in parentheses are these of C0 (C1) column sums. C0+C1 are numbers of Ac with C0 and C1 column sums. C0−C1 are numbers of Ac with C0 or C1.

the magnitude of the number of commodities effect is large. The similar results are obtained for row 2 and row 4 although the percentages are lower. The simulation results lead to the following conjecture.

CONJECTURE 2: *The more disaggregated input–output tables, the higher is the possibility of Ac having column sums under zero and/or over unity.*

REMARK 1: It is a well known fact that a proportion of secondary products of the total output of industries increases with diaggregation of input–output tables. See Stahmer (1982, p. 174) and United Nations (1973, p. 34). This seems to suggest the close correspondence between the conjecture 1 and conjecture 2.

3–2 Effects of $[s_L, s_U]$ on s_i

The simulation described above urged us to check the effects of a range of s_i on s_i*, because it may be unrealistic to suppose that s_i lies between 0 and 1. So, in the second simulation, by setting the lower limit s_L and the upper limit s_U, we take the random variable s_i to distribute uniformly over the $[s_U, s_L]$ to neglect the variable outside the interval.

We caried out 45 experiments for k=0.5, 0.7, 0.9, and (s_L, s_U)=(0.28, 0.72), (0.26, 0.74), . . . , (0.0, 1.0). For each set of values of k and (s_L, s_U), 10 random vectors and 20 random matrix C were generated. For each pair of s and C, the corresponding s_i* was calculated.

Figure 1 shows the experimental results. The proportions of C0 decrease with k. For k=0.5 the possibilities of C0 are strongly rising over the range from (0.24, 0.76) to (0.06, 0.94), and then close to unity; for instance, the per cent of C0 is only 0.9 per cent at (0.26, 0.74) and 77.0 percent at (0.10, 0.90). The dotted curves of C0+C1 lie entirely above the C0 curve. For k= 0.7 and 0.9 the results are similar although the rapid increases now occur at (0.18, 0.82) and (0.04, 0.96) respectively.

Thus, the graph reveals two important things. First, the possibilities of C0 and C0+C1 and are decreasing with k, suggesting the conjecure 1. Second, these posibilities are increasing with the range of s_i' s. This observation supports the following conjecture.

CONJECTURE 3: *The wider apart the range of s_i' s, the higher is the possibility of Ac with column sumu over unity and/or under zero.*

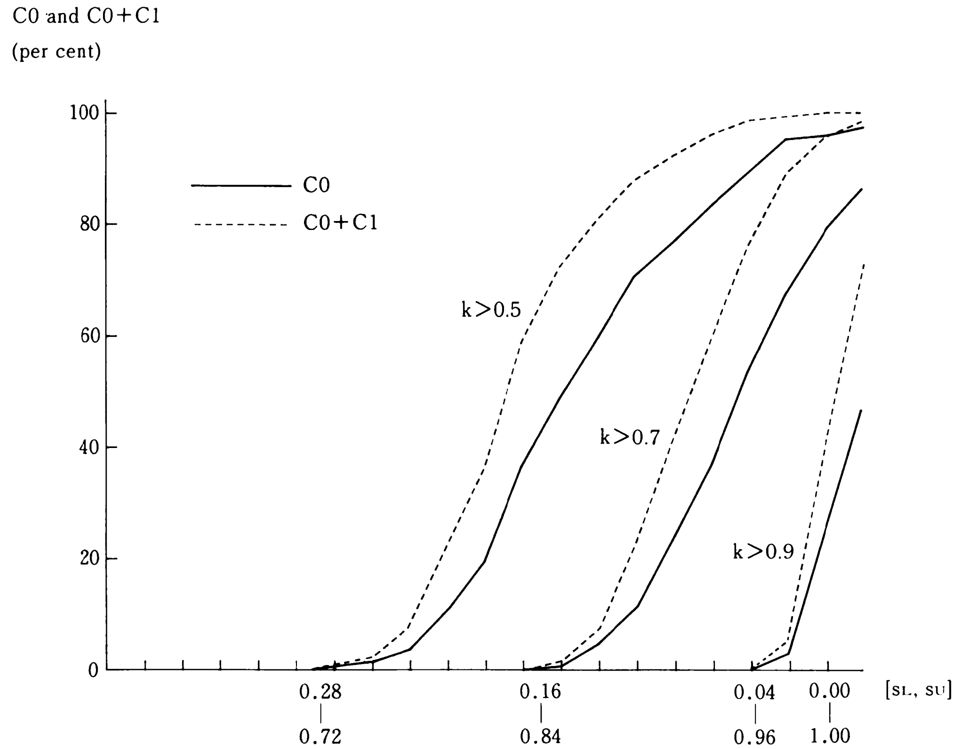

FIGURE 1 Column Sums of Ac and Range of $[s_L, s_U]$

4. Conclusion

We establish conjecture 1, 2 and 3 by the mathematical cosiderations and experimental results. We suggest that Ac is very risky in the sense that it will yield column sums under zero or over unity. These possibilities are especially true in the realistic case in which input—output k must be very large and the range of s must not be wide apart to justify the use of Ac. These fact cast doubt on the justification of Ac as theoretical sound from a computational ground. In fact cast doubt on the justification of Ac as theretical sound from a computational ground. In particular, our observation reveals the true reason why a great many countries have not taken the commodity technology assumption.

Department of Economics, Osaka Industrial University.

Footnotes

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2) Cressy (1976) also indicates this shortcoming (p. 128). ten Raa et al proposed the so—called by—product technology assumption, based on the commodity technology assumption. By treating commodity by—products as negative inputs, they successfully avoid the dependence on the choice of base—year prices.

REFERENCES

[1] Administrative Management Agency, Government of Japan (1984): *1980 Input–Output Tables*, Tokyo: Government of Japan.

[2] Cressy, R. C. (1976): "Commodity and Industry Technology: Symbols and Assumptions," *Manchester School of Economics and Social Studies*, 44, 112–131.

[3] Divay, J and F. Meunier (1982): "Two Methods of Elaborating Input–Output Tables," in *Compilation of Input–Output Tables*, ed. by J. Skolka. Berlin Heidelberg: Springer–Verlag.

[4] Geary, R. C. (1973): "Reflections on National Accounting," *The Review of Income and Wealth*, 19, 221–251.

[5] Gigantes, T. (1970): "The Representation of Technology in Input–Output Systems," in *Contributions to Input–Output Analysis*, ed. by A. P. Carter and A. Brody. Amsterdam: North–Holland.

[6] Olsen, J. A. (1984): "Adaptation of Detailed Input–Output Information: Restructuring and Aggregation," *The Review of Income and Wealth*, 31, 397–411.

[7] Price, G. B. (1951): "Bounds for Determinants with Dominant Principal Diagonal," *Proceedings of the American Mathematical Society*, 12, 497–502.

[8] ten Raa, Th., D. Chakraborty and J. A. Small (1984): "An Alternative Treament of Secondary Products in Input–Output Analysis," *Review of Economics and Statistics*, 66, 88–97.

[9] Stone, R., J. Bates and M. Bacharach (1963): *Input–Output Relations 1954–1966, A Programme for Growth 3*, London: Chapman & Hall.

[10] Stahmer, C. (1982): "Connecting National Accounts and Input–Output Tables in the Federal Republic of Germeny," in *Compilation of Input–Output Tables*, ed. by J. Skolka. Berlin Heidelberg: Springer–Verlag.

[11] Taussky,O. (1949): "A Recurring Theorem on Determinants," *American Mathematical Monthly*, 56, 672–676.

[12] Thage, B. (1982): "Techniques in the Compilation of Danish Input–Output Tables: A New Approach to the Treatment of Imports," in *Compilation of Input–Output Tables*, ed. by J. Skolka. Berlin Heidelberg: Springer–Verlag.

[13] United Nations (1968): *A System of National Accounts*, Series F, No.2, Rev. 3. New York.

[14] United Nations (1973): *Input–Output Tables and Analysis*. Series F, No. 14, Rev. 1. New York.